4.11 Solving Problems Using Quadratic Relations

What we have learned that we will be using:

- factoring and the quadratic formula leads to the roots

- finding the vertex (by factoring, partial factoring, or completing the square) gives you the optimal value

Remember that in word problems it is always important to identify the variables and sketching the parabola can be useful.

Ex.1 A hose is placed on an aerial ladder. The hose sprays water on a forest fire. The height of the water, *h*, in metres can be modelled by the relation: $h = -2.25(d - 1)^2 + 9$, where *d* is the horizontal distance, in metres, of the water from the nozzle of the hose.

a) What is the maximum height reached by the water?

∴ the max height reached by the water was 9 m

b) At what horizontal distance from the nozzle is the maximum height reached?

∴ the horizontal distance from the nozzle is 1 m

c) What is the height of the aerial ladder?

$d = 0$

$h = -2.25(0-1)^2 + 9$

$= 6.75$

∴ the height of the aerial ladder is 6.75 m.

d) How high is the water when it is at a horizontal distance of 2 m from the nozzle?

$= -2.25(2-1)^2 + 9$

$= 6.75$

∴ the height of the water at $d = 2$ is 6.75 m

Ex.2 A ball is thrown into the air. Its height, in metres, after *t* seconds, is $h = -4.9t^2 + 39.2t + 1.75$.

a) When does it reach maximum height?

$h = -4.9(t^2 - 8t) + 1.75$

$= -4.9((t^2 - 8t + 16) - 16) + 1.75$

$= -4.9[(t-4)^2 - 16] + 1.75$

$= -4.9(t-4)^2 + 78.4 + 1.75$

$= -4.9(t-4)^2 + 80.15$

∴ it reach its max height at 4 sec.

b) What is the maximum height?

Ex.2 A ball is thrown into the air. Its height, in metres, after *t* seconds, is $h = -4.9t^2 + 39.2t + 1.75$.

c) From what height is the ball released? $t = 0$

∴ the ball was released from 1.75 m.

d) When does the ball hit the ground?

$0 = -4.9(t-4)^2 + 80.15$

$\frac{-80.15}{-4.9} = \frac{-4.9(t-4)^2}{-4.9}$

$\sqrt{16.35714} = \sqrt{(t-4)^2}$

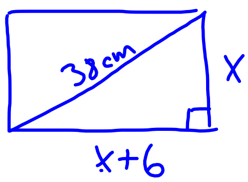
$\pm 4.04 = t - 4$

$4 \pm 4.04 = t$

$x_1 \doteq 8.04$ $x_2 \doteq -0.04$ Reject!

∴ the ball hits the ground after 8.04 sec.

Ex.3 The size of a television screen or computer monitor is usually stated as the length of the diagonal. A screen has a 38-cm diagonal. The width of the screen is 6 cm more than the height. Find the dimensions of the screen to the nearest tenth.



$a^2 + b^2 = c^2$

$x^2 + (x+6)^2 = 38^2$

$x^2 + (x+6)^2 - 38^2 = 0$

$x^2 + (x+6)(x+6) - 38^2 = 0$

$x^2 + x^2 + 12x + 36 - 38^2 = 0$

$2x^2 + 12x + 36 - 1444 = 0$

$2x^2 + 12x - 1408 = 0$

a b c

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(12) \pm \sqrt{(12)^2 - 4(2)(-1408)}}{2(2)}$

$x_1 = 23.70$

~~$x_2 = -29.7$~~

∴ the width is 29.70 cm & height is 23.70 cm

Ex. 4 A sporting goods store sells 90 ski jackets in a season for $200 each. Each $10 decrease in price would result in five more jackets being sold.

(a) Find the number of jackets sold and the selling price to give maximum revenues.

(b) What is the lowest price that would produce revenues of at least $15600?

(c) How many jackets would be sold at this price?

$R=(90+5x)(200-10x)$

$5x=-90$ or $-10x=-200$

$x=-18$ $x=20$

$AOS=\frac{-18+20}{2}$

$x=1$

sub $x=1$ into eq'n

$R=(90+5(1))(200-10(1))$

$=18050$

$15600=(200-10x)(90+5x)$

$15600=18000+1000x-900x-50x^2$

$0=-50x^2+100x+2400$

$=-50(x^2-2x-48)$

$=-50(x+6)(x-8)$

~~$x=-6$~~ or $x=8$

Assigned Work: p. 357 #2, 3, 5, 7, 9, 14